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Consideration of Plant Uncertainties in the Optimum Structural-Control Design

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An integrated design approach for the optimum design of a controlled structure is presented. The control design method takes into consideration both structured and unstructured uncertainties by improving a bound on the H_∞ norm of the closed-loop system and satisfying a constraint on the linear quadratic Gaussian performance index. The controller is designed by solving three Riccati equations. The weight of the structure is specified as an objective function with constraints on the structural frequencies, the closed-loop damping, and the performance index. The numerical results are presented for a three-dimensional truss structure to illustrate the application of the integrated approach.

Introduction

THE integrated design approach for controlled structures requires a formulation of the problem that takes into consideration the design requirements of the structure as well as the control so that the overall system performs at peak efficiency. The structure is required to be of minimum weight, and the control system needs to be robust. In the case of a flexible structure, it is difficult to separate the two requirements without meaningful tradeoff. This tradeoff can be achieved only by using these two disciplines simultaneously in the design process and defining it as a mathematical optimization problem. The structure must satisfy constraints on stresses, displacements, frequency distribution, etc., and the control system must be robust and have good performance. The control system is said to be robust if it can maintain stability and performance in the presence of plant uncertainties. The plant for the design of a control system for a structure is a function of structural frequencies, damping, and vibration modes. The uncertainties can be generally characterized into two groups: structured and unstructured. Structured uncertainty is defined as the variation of the real parameters in the plant. This may be due to the inaccuracies in the calculation of the frequencies and damping due to the approximations in the structural model, material properties, mass, damping, etc. The unstructured uncertainty is due to the neglect of actuator and sensor dynamics and higher-order structural modes. This paper takes into consideration these uncertainties in structural and control design.

Several researchers have treated the simultaneous design of the structure and control systems in recent years. Hale[1] and Salama et al.[2] have minimized the sum of the mass of the structure and quadratic control cost function with constraints on the structural and control design variables. The sensitivity of the control system to the structural modifications has been investigated by Haftka et al.[3] The integrated approach with the weight of the structure as the objective function and constraints on the damping parameters and frequency distribution of the closed-loop system was presented by Khot.[4] Simultaneous optimization with constraints on the closed-loop eigenvalues and Frobenious norm was considered by Khot et al.[5] The multiobjective problem of the structure and control design was discussed by Rao.[6] Lust and Schmit[7] considered minimization of the structural mass and performance index with behavior constraints on displacements and frequencies. The robustness of the control system was not addressed in these publications. Lim and Junkins[8] considered structural and control design by utilizing robustness parameters based on the solution of the Lyapunov equations. Rew et al.[9] presented pole-placement techniques for obtaining robust eigenstructure assignment. Khot and Veley[10] used robustness parameters based on the spectral radius of the matrix in integrated structure control problems. These approaches primarily account for only structured uncertainties.

The purpose of this paper is to develop an algorithm to design the structural and control systems simultaneously, yielding a structure that is of minimum weight and a controller that is robust under both structured and unstructured uncertainties. In the definition of the optimization problem, the weight of the structure is considered as the objective function and the constraints are imposed on the distribution of the structural frequencies, the upper bound on the linear quadratic Gaussian (LQG) performance index, and the closed-loop damping. The control design procedure is based on designing the LQG compensator with a constraint on the H_∞ norm of the closed-loop system. The compensator satisfying these requirements is found by solving three Riccati equations and is based on the approach proposed in Ref. 11. The application of this integrated design approach is demonstrated on a three-dimensional truss problem.

Control Design

The equations of motion of a flexible structure in state vector form can be written as

$$\dot{x} = \bar{A}x + \bar{B}f \tag{1}$$

where $x(n_1 \times 1)$ is the state vector, $f(p \times n_1)$ is the control input vector, $\bar{A}(n_1 \times n_1)$ is the plant matrix, and $\bar{B}(n_1 \times p)$ is the control input matrix. The plant and input matrices for a structural problem are given by

$$\bar{A} = \left[\begin{array}{c|c} 0 & I \\ \hline -\omega^2 & -2\zeta\omega \end{array}\right] \tag{2}$$

$$\bar{B} = \left[\begin{array}{c} 0 \\ \hline \phi^T D \end{array}\right] \tag{3}$$

where ω^2 is a diagonal matrix of the squares of the structural frequencies, ζ is the vector of modal structural damping,

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$-2\zeta\omega$ is a diagonal matrix, and D is the applied actuator load distribution matrix. Equation (1) is written for a full-order system or reference system. The number of state variables associated with this system is larger than the number of state variables used to design the control system. The number of state variables chosen for the control design is substantially smaller than n_1.

The control design approach used in this paper utilizes the simultaneous linear quadratic Gaussian (LQG) and H_∞ optimization. The dynamic equations of motion for an uncertain reduced-order system can be written as

$$\dot{x} = (A + \Delta A)x + Bf \tag{4}$$

$$y = Cx + w_2 \tag{5}$$

$$z_1 = Cx \tag{6}$$

where x is an n-dimensional state vector, f is a p-dimensional input vector, y is a q-dimensional output vector, and z_1 is a q-dimensional measured output vector. $A(n \times n)$, $B(n \times p)$, and $C(p \times n)$ are plant, input, and output matrices of the reduced-order model ($n_1 > n$). In Eq. (5), w_2 is a p-dimensional disturbance vector. The matrix ΔA is a real-parameter variation matrix, which is unknown but assumed to lie within some interval. Equations (4) and (5) with the fact that $C = B^T$ indicate that actuators and sensors are colocated. The nth-order compensator represented by the transfer function $K(s)$ can be written as

$$\dot{x}_c = A_c x_c + B_c y \tag{7}$$

$$f = C_c x_c \tag{8}$$

Using Eqs. (4–8), the closed-loop system equations can be written as

$$\dot{x} = (\underline{A} + \Delta\underline{A})x + \underline{D}w \tag{9}$$

$$z_1 = \underline{C}x \tag{10}$$

where

$$x = \begin{bmatrix} x \\ x_c \end{bmatrix}, \qquad w = \begin{bmatrix} 0 \\ w_2 \end{bmatrix} \tag{11}$$

$$A = \begin{bmatrix} A & BC_c \\ B_c C & A_c \end{bmatrix} \tag{12}$$

$$\Delta\underline{A} = \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix} \tag{13}$$

$$\underline{D} = \begin{bmatrix} 0 & 0 \\ 0 & B_c \end{bmatrix} \tag{14}$$

and

$$\underline{C} = [C \quad 0] \tag{15}$$

where $\underline{A}(2n \times 2n)$, $\Delta\underline{A}(2n \times 2n)$, $\underline{D}(2n \times 2n)$, and $C(p \times 2n)$ are closed-loop matrices. The transfer function between z_1 and w_2 (see Fig. 1) can be written as

$$H(s, \Delta A) = G(s, \Delta A)K(s)[I + G(s, \Delta A)K(s)]^{-1} \tag{16}$$

where

$$G(s, \Delta A) = C(sI - A - \Delta A)^{-1}B \tag{17}$$

$G(s, \Delta A)$ represents the transfer function of the uncertain plant, and $K(s)$ is the transfer function of the compensator given by

$$K(s) = C_c(sI - A_c)^{-1}B_c \tag{18}$$

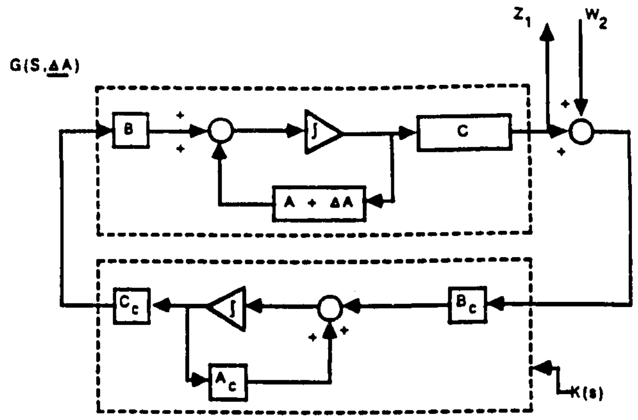

Fig. 1 Close-loop system for robust control.

The control design problem can be stated as follows: For a given state space, A, B, C, and ΔA matrices determine the controller state space matrices, A_c, B_c, and C_c such that the following conditions are specified.

1) The closed-loop system is asymptotically stable.
2) The H_∞ norm of Eq. (16)

$$\|H(s, \Delta A)\| \le \gamma \tag{19}$$

where γ is a specified parameter.

3) The linear quadratic Gaussian performance index satisfies the condition

$$J(\Delta A) = \lim E[x^T R_1 x + u^T R_2 u] \le J_u \tag{20}$$

where J_u is the least upper bound on J. J_u depends on the weighting matrices R_1 and R_2 and the controller matrices A_c, B_c, and C_c. For specified γ, the controller matrices A_c, B_c, and C_c are calculated by solving three coupled Riccati matrices. Given a system with unmodeled dynamics $L(s)$ modeled as multiplicative perturbations at the plant output, the H_∞ norm of Eq. (16) is inversely proportional to the level of the unmodeled dynamics $L(s)$ that the controller can accommodate. Thus, the smaller the value of γ for which the controller can be designed, the higher the unmodeled dynamics that can be tolerated. However, for small value of γ it is numerically difficult to obtain the controller state space matrices satisfying all of the Riccati equations.

Numerical Procedure for Control Design

The specified model parameter variation matrix ΔA is first paramaterized as

$$\Delta A = \sum_{i=1}^{p_2} D_i \underline{M}\underline{N} E_i \tag{21}$$

where D_i and E_i are the column and row vectors, respectively, of dimension n, which describe the structure of the uncertainty. D_i contains only nonzero elements characterizing the ith parameter variation, and E_i is the nonzero element equal to unity at a location corresponding to the nonzero element in D_i (transpose). In Eq. (21), p_2 is the number of terms to be parameterized and the product $\underline{MN}$ is a scalar matrix equal to the identity matrix. The iterative procedure for designing the controller consists of the following steps:

1) For specified values of $\underline{M}$ and $\underline{N}$ compute

$$V_1 = \underline{M}\sum_{i=1}^{p_2} D_i D_i^T, \qquad V_2 = \underline{N}\sum_{i=1}^{p_2} E_i^T E_i \tag{22}$$

2) Select the control weighting matrix R_2 and the parameter γ and compute

$$R_{1\infty}^o = R_1 = C^T C + \gamma^2 V_2 \tag{23}$$

$$R_{q1\infty} = \gamma^{-2} R_1 \tag{24}$$

3) Solve for the symmetric positive definite Riccati matrix Q from the equation

$$A_q Q + Q A_q^T + Q R_q Q + V_q = 0 \tag{25}$$

where

$$A_q = A \tag{26}$$

$$R_q = R_{q1\infty} - C^T C \tag{27}$$

$$V_q = V_1 \tag{28}$$

4) Compute

$$B_c = B_{co} = Q C^T \tag{29}$$

and initialize $C_c = C_{co} = R_2^{-1} B^T$.

5) Solve for a symmetric positive definite Riccati matrix $\underline{Q}$ from the equation

$$A_{q1} \underline{Q} + \underline{Q} A_{q1}^T + \underline{Q} R_{q1\infty} \underline{Q} + V_{q1} = 0 \tag{30}$$

where

$$A_{q1} = A + Q\gamma^{-2} R_{1\infty}^{o} + B C_c \tag{31}$$

$$V_{q1} = B_c B_{co}^T \tag{32}$$

and $R_{q1\infty}$ is computed in step 2.

6) Solve for a symmetric positive definite Riccati matrix P from the equation

$$A_p^T P + P A_p + P R_p P + R_1 = 0 \tag{33}$$

where

$$A_p = A + (Q + \underline{Q}) R_{q1\infty} \tag{34}$$

$$R_p = -B R_2^{-1} B^T \tag{35}$$

and R_1 is computed in step 2.

7) Compute

$$\epsilon = \sigma_{\max}(C_c - C_{co} P) \tag{36}$$

where $\sigma_{\max}$ denotes the maximum singular value. Singular values are a measure of the size of the system matrix, and maximum singular values give an indication of how much amplification a particular transfer function under consideration will produce on a given signal of disturbance. If ϵ is smaller than the specified convergence criteria, then go to step 8; otherwise set $C_c = C_{co} P$ and repeat steps 5–7.

8) For the converged solution compute

$$A_c = A_{q1} - B_c C \tag{37}$$

$$B_c = Q C^T \tag{38}$$

$$C_c = C_{co} P \tag{39}$$

$$J_u = \text{tr}\,(Q + \underline{Q}) R_1 + \text{tr}\,(\underline{Q} C_c^T R_2 C_c) \tag{40}$$

and calculate $\|H(s)\|_\infty$ norm of

$$H(s) = G(s) K(s) [I - G(s) K(s)]^{-1} \tag{41}$$

where

$$G(s) = C(sI - A)^{-1} B \tag{42}$$

and

$$K(s) = C_c (sI - A_c)^{-1} B_c \tag{43}$$

If the calculated norm is larger than the desired value, then reduce γ and repeat steps 2–8. This process may be continued until the converged solution can be obtained for the smallest γ. The limiting value of γ is one.

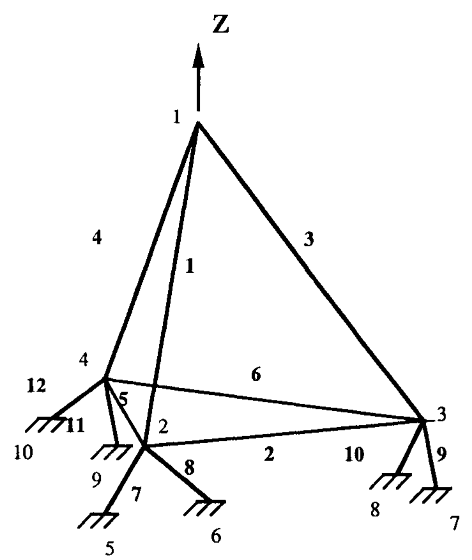

Fig. 2 Truss structure.

Table 1 Design variables

Element number	Design 1	Design 2	Design 3
1 (1–2)	1000.0	262.55	163.91
2 (2–3)	1000.0	249.14	192.18
3 (1–3)	100.0	173.82	134.82
4 (1–4)	100.0	123.53	173.76
5 (2–4)	1000.0	241.75	196.42
6 (3–4)	1000.0	256.80	196.26
7 (2–5)	100.0	59.95	72.11
8 (2–6)	100.0	128.98	49.21
9 (3–7)	100.0	101.81	314.41
10 (3–8)	100.0	76.63	99.53
11 (4–9)	100.0	125.35	165.31
12 (4–10)	100.0	52.45	231.89
Weight	43.69	14.61	13.21
$\bar{\gamma}$	1.0	0.666	0.162

Integrated Design Procedure

In the present problem, the integrated design process is formulated as a mathematical optimization problem and the objective function to be minimized is the weight of the structure. The problem can be stated as follows:

Minimize the weight:

$$W = \sum_{i=1}^{\bar{n}} \rho_i A_i \ell_i \tag{44}$$

subject to the structural constraints

$$g_j(\omega_i^2) = \omega_i^2 - \bar{\omega}_i^2 \geq 0 \tag{45}$$

and the control design constraints

$$g_j(J_u) = \bar{J}_u - J_u \geq 0 \tag{46}$$

$$g_j(\xi_i) = \xi_i - \bar{\xi}_i \geq 0 \tag{47}$$

In Eq. (44), ρ_i denotes the density of the material, l_i is the length of the element, and A_i is the cross-sectional area of the element. The number of structural design variables is equal to $\bar{n}$. Equation (45) defines the constraint on the square of the structural frequency. The lower bound on the square of the structural frequencies is $\bar{\omega}_i^2$. The constraint on the upper bound of the linear quadratic Gaussian performance index

Table 2 Constraint values

$\gamma = 30$	
Initial values	Final values
$\omega_1^2 = 1.801$	$\omega_1^2 = 1.801$
$J_u = 110.2$	$J_u = 79.64$
$\xi_1 = 0.070$	$\xi_1 = 0.085$
Weight = 43.69	Weight = 14.61
$\gamma = 3$	
Initial values	Final values
$\omega_1^2 = 1.801$	$\omega_1^2 = 1.801$
$J_u = 2.095$	$J_u = 1.135$
$\xi_1 = 0.0515$	$\xi_1 = 0.0625$
Weight = 43.69	Weight = 13.21

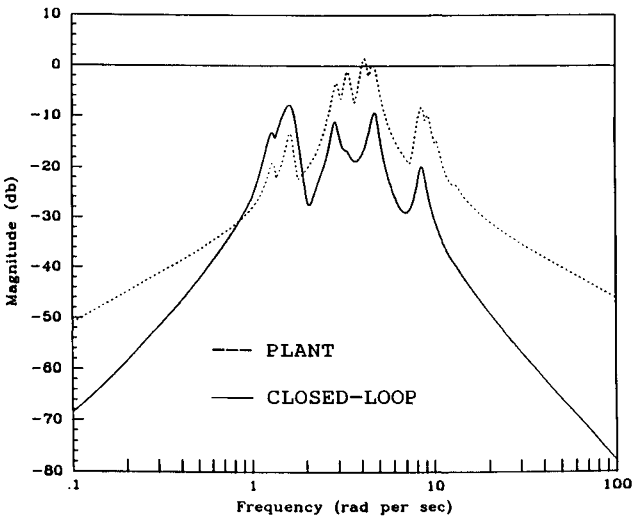

Fig. 3 Maximum singular values initial design, $\gamma = 30$.

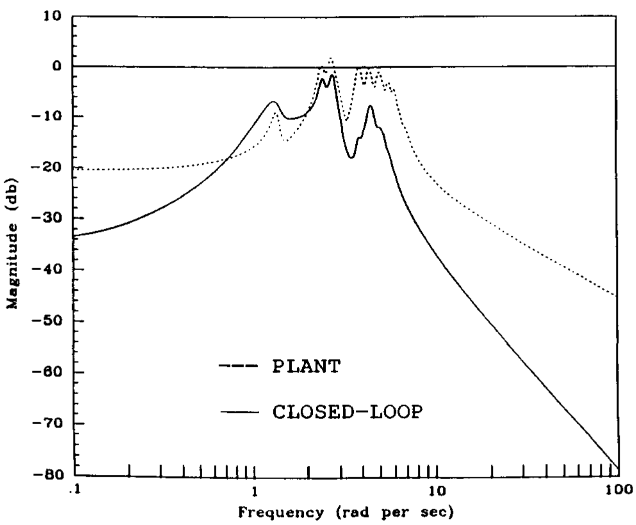

Fig. 4 Maximum singular values optimum design, $\gamma = 30$.

[Eq. (40)] is defined in Eq. (46). $\bar{J}_u$ is the maximum allowable value of the upper bound on the performance index. The constraints on the closed-loop damping of matrix $\underline{A}$ [Eq. (12)] are defined in Eq. (47) where $\bar{\xi}_i$ is the minimum acceptable value. In addition, constraints were imposed on the minimum values of the structural design variables. The integrated optimization problem was solved by using the NEWSUMT-A[12] mathematical optimization program based on the extended interior penalty function method with Newton's method of unconstrained minimization.

Numerical Example

The truss structure[10] shown in Fig. 2 was designed with specified constraints. This structure is flexible and fixed at the base. The dimensions and elastic properties are specified in nondimensional units. The Young's modulus is equal to unity, and the density of the material is assumed to be 0.1. The structure has 12 elements and 12 deg of freedom. Six colocated actuators and sensors are in elements 7–12. The state-space model for the full-order system corresponding to Eq. (1) will be 24×24. The number of states in this equation is 24, and the number of inputs is six. Thus, the input matrix $\bar{B}$ and output matrix $\bar{C}$ is 24×6 and 6×24, respectively. For the reduced-order model, only the first two lowest structural frequencies were used, and the state-space model was formulated by truncating all higher-order modes. Thus, the matrices A, B, and C in Eqs. (4–6) would then be 4×4, 4×6, and 6×4, respectively. The cross-sectional areas for the nominal design or the initial design for optimization, designated as design 1, are given in Table 1.

Initially, the compensator was designed for design 1 for various values of γ to study the convergence of the algorithm and the minimum value of γ for which the compensator can be designed that satisfies all of the conditions on the Riccati

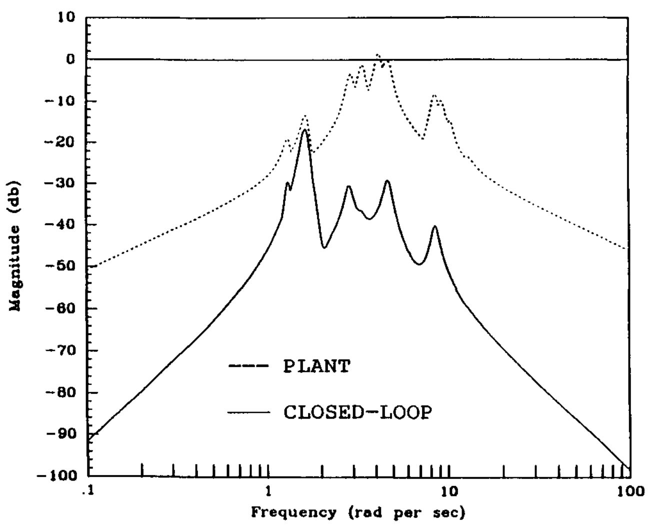

Fig. 5 Maximum singular values initial design, $\gamma = 3$.

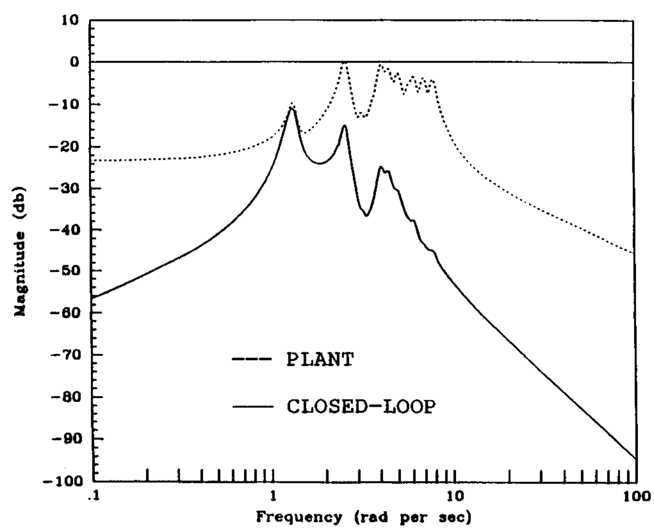

Fig. 6 Maximum singular values optimum design, $\gamma = 3$.

equations and the convergence criterion on the singular value defined in Eq. (36). The value of ϵ selected for this purpose was 0.0001. The weighting matrix $\underline{M}$ was a scalar quantity set equal to 110. The percentage uncertainty in the elements of the plant matrix was set equal to $\pm 5\%$ for calculation of matrix ΔA. The lowest value of γ was found to be 2.6. However, this needed a large number of iterations in order to satisfy the convergence criterion. Since $\gamma = 2.6$ during optimization might force the design to enter into the nonconvergent region of the control design space of the compensator, a slightly higher value of $\gamma = 3.0$ was used. During optimization the value of γ was fixed. The results presented here are for $\gamma = 30$ and $\gamma = 3$, respectively.

The optimization problem was set up to satisfy the following constraints:

$$\omega_1^2 \geq \bar{\omega}_1^2 \tag{48}$$

$$\xi_1 \geq \bar{\xi}_1 \tag{49}$$

$$J_u \leq \bar{J}_u \tag{50}$$

where $\bar{\omega}_1^2$, $\bar{\xi}_1$, and $\bar{J}_u$ represent the square of the lowest structural frequency, damping associated with the lowest frequency of the closed-loop system [Eq. (12)], and upper bound on the performance index [Eq. (46)] for the nominal design, respectively.

The optimization program NEWSUMT-A required 12 iterations to obtain an optimum design for $\gamma = 3$. All constraint sensitivities were calculated by the finite difference approach. The design parameters were the cross-sectional areas of the members and a parameter $\bar{\gamma}$ that multiplied the identity matrix to compute the control weighting matrix R_2. The values of the design variables for $\gamma = 30$ (design 2) and $\gamma = 3$ (design 3) are given in Table 1. For optimization, $\bar{\omega}_1^2$ was set to the square of the lowest frequency for the initial design. The constraint on the performance index $\bar{J}_u$ was also equal to the initial value. The constraint value $\bar{\xi}_1$ on the closed-loop damping was set equal to 1.2 times that of the initial design. The initial and final values of the constraints are given in Table 2. The initial weight of the structure was 43.69 units and the optimum weights for $\gamma = 30$ and $\gamma = 3$ were 14.61 and 13.21 units, respectively.

The control analysis on the initial and optimum designs was performed using the original full-order model as a measure of robustness to the truncated higher order dynamics. Maximum singular values were computed for both plant $G(s)$ and the closed-loop system $G(s)K(s)[I + G(s)K(s)]^{-1}$ at frequencies from 0.01 to 100 rad/s for all designs.

Figures 3 and 4 show open- and closed-loop maximum singular values for the initial and optimum design for $\gamma = 30$. Here, the closed-loop curve crosses the open-loop curve indicating that the controller's ability to suppress the structure's first modes actually decreases in the closed loop. The controller has actually a destabilizing effect. The singular-value plots for the initial and optimum designs for $\gamma = 3$ are shown

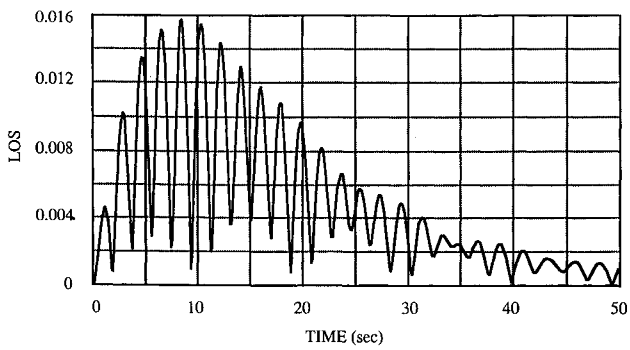

Fig. 7 Transient response for initial design, $\gamma = 3$.

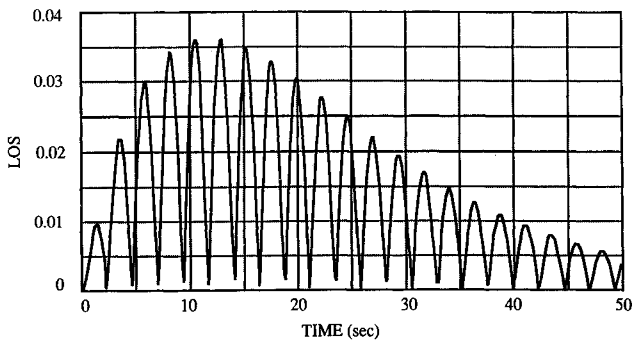

Fig. 8 Transient response for optimum design, $\gamma = 3$.

in Figs. 5 and 6, respectively. For both designs, the closed-loop response in general shows the controller attenuates each of the structural modes of the analysis model, particularly the higher-order modes. Since the amount of unmodeled dynamics the controller can tolerate is proportional to the inverse of H_∞ norm of Eq. (16), the optimum design 3 obtained with $\gamma = 3$ would provide higher robustness to the unmodeled dynamics.

The dynamic response of the closed-loop system for the design with $\gamma = 3$ was determined as the motion of node 1 or the apex in the *x-y* plane. The radial displacement was computed as equal to the square root of the sum of the x component and y component squared of the apex movement. An impulse force was applied to actuator 1 located in element 7 at time $t = 0$, and the transient response was then computed as shown in Figs. 7 and 8 for the initial and optimum design for a period of 50 s. The controller designed on the basis of the first two structural modes was used to control the full-order system. For both the initial and optimum designs, the closed-loop responses required approximately the same amount of time to settle. However, the peak magnitude of the response for the optimum design was higher than that for the initial design. A better response of the optimum design could have been obtained by increasing the number of constraints on the closed-loop damping.

Summary

In this paper, the integrated control and structure design optimization problem has been investigated by using a control approach that takes into consideration structured and unstructured uncertainties in the plant matrix. The control approach is based on designing the LQG compensator with constraint on the H_∞ norm of the closed-loop system. The algorithm was used to design a truss structure with a 24×24 plant matrix to be controlled by a controller based on 4×4 plant matrix. The truncation of the full-order system was achieved by direct truncation of the higher modes. The objective function was the weight of the structure with constraints on the structural frequency closed-loop damping of the lowest frequency and upper bound on the closed-loop performance index. The system was designed for two values of parameters γ, which bounds the H_∞ norm. Singular values were computed for the open-loop and closed-loop systems for a comparison of both cases. The transient response of the optimum and initial designs were investigated.

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